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Non-Leptonic Heavy Meson Decays – Theory Status

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I briefly review the status and recent progress in the theoretical understanding of non-leptonic decays of beauty and charm hadrons. Focusing on a personal selection of topics, this covers perturbative calculations in quantum chromodynamics, analyses using flavour symmetries of strong interactions, and the modelling of the relevant hadronic input functions.

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1 Motivation

Non-leptonic decays of hadrons containing a heavy bottom or charm quark may provide important information on the angles of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the Standard Model (SM). They also may reveal deviations from the SM, in particular the presence of new CP-violating phases from “new physics” (NP). The non-trivial hadronic dynamics in such flavour transitions further allows to assess the accuracy of theoretical methods (perturbative or non-perturbative) in Quantum Chromodynamics (QCD), based on the separation (“factorization”) of short- and long-distance strong-interaction effects. Finally, comparison of experimental data and theoretical parametrizations may also lead to a better understanding of the hadronic structure of heavy-light bound states.

Experimental studies of non-leptonic B - and D -meson decays, which have been successfully carried out at flavour factories and at hadron colliders in the past, will be continued at present and future experiments, notably at LHCb and Belle-II. With the foreseen increasing experimental precision, theoretical calculations should therefore catch up in accuracy in order to achieve reliable phenomenological conclusions about the validity of the SM or hints for NP (for comprehensive reviews, see e.g. [1–4]).

In this proceedings contribution, I highlight some recent theoretical results and developments which contribute to improving our understanding of exclusive heavy-meson decays.

2 Perturbative Calculations

Theory predictions for non-leptonic exclusive decays, by definition, depend on hadronic matrix element which cannot be calculated in QCD perturbation theory. Still, the presence of a heavy quark mass (notably for the b -quark) implies that certain dynamical effects are related to short-distance physics (on length scales of the order $1/m_b$) and may thus be accessible in perturbative QCD. The challenge is then to systematically separate short- and long-distance phenomena, where the latter should be described by as few independent hadronic parameters as possible.

QCD Factorization for B Decays into Light Mesons

B -decays into two (energetic) light mesons (M_1, M_2) are described by hadronic matrix elements of weak transition operators \mathcal{O}_i . In the limit of infinitely heavy quark masses, the strong-interaction dynamics factorizes according to [5]

$$\begin{aligned} \langle M_1 M_2 | \mathcal{O}_i | B \rangle &\simeq F^{B \rightarrow M_1} \int du T_i^I(u) \phi_{M_2}(u) \\ &+ \int d\omega du dv T_i^{\text{II}}(\omega, u, v) \phi_B(\omega) \phi_{M_1}(v) \phi_{M_2}(u). \end{aligned} \quad (1)$$

Here, the hadronic input functions are given by (universal) transition form factors $F^{B \rightarrow M}$ evaluated at large recoil energy, and light-cone distribution amplitudes (LCDAs) ϕ_{B,M_1,M_2} for heavy and light mesons which depend on the momenta (respectively momentum fractions) of the light quarks. The short-distance kernels $T_i^{\text{I,II}}$ can be calculated perturbatively, including renormalization-group (RG) improvement in the framework of soft-collinear effective theory (SCET [6, 7]). One generic phenomenological consequence of this “QCD-improved” factorization (QCDF) is that direct CP violation in these decays – which requires strong rescattering phases – is suppressed by $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(1/m_b)$ in the heavy-quark limit.

The result of QCDF calculations can be parametrized in terms of (decay-channel dependent) tree- and penguin-amplitude parameters α_i , for instance,*

$$\begin{aligned} \langle \pi^+ \pi^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle &= A_{\pi\pi} \{ \lambda_u [\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)] + \lambda_c \alpha_4^c(\pi\pi) \} \\ \langle \pi^+ K^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle &= A_{\pi K} \{ \lambda_u^{(s)} [\alpha_1(\pi K) + \alpha_4^u(\pi K)] + \lambda_c^{(s)} \alpha_4^c(\pi K) \} \\ &\text{etc.} \end{aligned} \quad (2)$$

The current status of higher-order calculations (NNLO, i.e. second order in the strong coupling α_s) is as follows.

- 2-loop vertex corrections contributing to T_i^{I} for tree-amplitude parameters $\alpha_{1,2}$ have been determined independently in [9, 10]. The corresponding 2-loop vertex corrections for penguin amplitudes are currently under study [11], and preliminary results will be shown below.
- The 1-loop spectator corrections to T_i^{II} for tree amplitudes have been analyzed in [13–15]. The 1-loop spectator corrections for penguin amplitudes can be found in [16, 17].

Let us briefly discuss the numerical significance of the individual contributions. Considering e.g. the colour-allowed (-suppressed) tree amplitude $\alpha_1(\alpha_2)$ in $B \rightarrow \pi\pi$ decays, we have

$$\begin{aligned} \alpha_1(\pi\pi) &= [1.008]_{V_0} + [0.022 + 0.009i]_{V_1} + [0.024 + 0.026i]_{V_2} \\ &\quad - [0.014]_{S_1} - [0.016 + 0.012i]_{S_2} - [0.008]_{1/m_b} \\ &= 1.015_{-0.029}^{+0.020} + (0.023_{-0.015}^{+0.015}) i, \end{aligned} \quad (3)$$

and

$$\begin{aligned} \alpha_2(\pi\pi) &= [0.224]_{V_0} - [0.174 + 0.075i]_{V_1} - [0.029 + 0.046i]_{V_2} \\ &\quad + [0.084]_{S_1} + [0.037 + 0.022i]_{S_2} + [0.052]_{1/m_b} \end{aligned}$$

*Here λ_q denote combinations of CKM elements, and the normalization factors $A_{M_1 M_2}$ are given in terms of form factors and decay constants [8].

$$= 0.194_{-0.095}^{+0.130} - \left(0.099_{-0.056}^{+0.057}\right) i, \quad (4)$$

where $V_{0,1,2}$ stand for (LO,NLO,NNLO) contributions to T_i^{I} , while (NLO,NNLO) spectator contributions from T_i^{II} are labeled by $S_{1,2}$. Estimates of $1/m_b$ power corrections are also quoted. One observes that

- the perturbative expansion is well behaved, with individual NNLO corrections V_2 and S_2 being significant but tending to cancel in the sum;
- precise predictions are achieved for the colour-allowed tree amplitude α_1 , while larger hadronic uncertainties remain for the colour-suppressed amplitude α_2 ;
- the relative phase between α_1 and α_2 stays small at NNLO.

Penguin amplitudes are currently known at NLO, and for the $\pi\pi$ channel one gets

$$\begin{aligned} \alpha_4^u(\pi\pi) &= -0.024_{-0.002}^{+0.004} + \left(-0.012_{-0.002}^{+0.003}\right) i \\ \alpha_4^c(\pi\pi) &= -0.028_{-0.003}^{+0.005} + \left(-0.006_{-0.002}^{+0.003}\right) i \end{aligned} \quad (5)$$

The calculation of penguin amplitudes at NNLO, which is currently worked out [11], involves $\mathcal{O}(70)$ 2-loop diagrams with up to 3 independent mass scales (m_b, m_c, um_b) and 4 external legs and non-trivial charm thresholds at $(1-u)m_b^2 = 4m_c^2$ (where u is the momentum fraction of a quark in a light meson). Preliminary results have been shown in recent conference talks [12].

In the past, the QCDF results have been used for comprehensive phenomenological studies:

- The decays $B \rightarrow \pi\pi, \pi\rho, \rho\rho$ have been investigated in [10, 18]. Predictions for colour-suppressed modes are rather uncertain and typically underestimated in QCDF (depending on the hadronic matrix element governing the size of spectator-scattering contributions). The uncertainties from hadronic form factors and the CKM element $|V_{ub}|$ can be reduced by considering ratios with the semi-leptonic rates.
- Estimates for tree-dominated B_s decays can be found in [19]. Here the relevant hadronic parameters (form factors and LCDAs) are less well known than for the previous case. On the other hand, the pattern of annihilation contributions is simpler. It has also been emphasized that the size of charming-penguin effects can be tested from ratios of colour-allowed modes [20].
- One can also find results for charmless B -meson decays into scalar mesons [21].

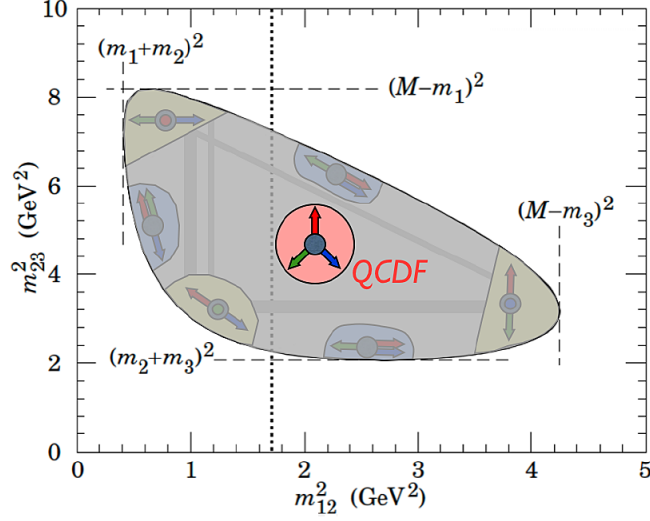


Figure 1: Dalitz plot of $B \rightarrow \pi\pi\pi$ decays, highlighting the “mercedes-star” configuration where QCDF is applicable (from [24]).

Other modes / Further Activities

NNLO corrections in QCDF for the decays $B \rightarrow D\pi$ are currently studied as well. In this case, the charm and bottom quark are usually treated as heavy quarks in HQET, and their hard fluctuations are integrated out at a common matching scale. Compared to charmless decays, this leads to a number of new master integrals which depend on the mass ratio m_c^2/m_b^2 . The status of the computation has been recently reported in [22].

The systematics of QCDF in e.g. $B \rightarrow \pi\pi$ decays can also be independently addressed by phenomenological studies of the related decay $B \rightarrow \pi\pi\ell\nu$. In the region where the dipion invariant mass is large, $m_{\pi\pi}^2 \sim \mathcal{O}(m_b^2)$, one obtains a factorization theorem [23] which takes a similar form as in $B \rightarrow \pi\pi$ (with \otimes denoting the convolution integrals as in (1)),

$$\langle \pi^+ \pi^- | \bar{u} \Gamma b | B \rangle \simeq F^{B \rightarrow \pi} \cdot \phi_\pi \otimes T_\Gamma^{\text{I}}(q^2) + \phi_\pi \otimes \phi_\pi \otimes \phi_B(\omega) \otimes T_\Gamma^{\text{II}}(q^2). \quad (6)$$

In contrast to $B \rightarrow \pi\pi$, the decay is now induced by semi-leptonic operators, where $\Gamma = \gamma_\mu(1 - \gamma_5)$ in the SM. In the considered kinematic region, at least one hard gluon is required to produce the additional back-to-back quark-antiquark pair in the final state. As a consequence, the kernel T_Γ^{I} starts at $\mathcal{O}(\alpha_s)$, while the kernel T_Γ^{II} includes additional spectator interactions and thus starts at $\mathcal{O}(\alpha_s^2)$. Moreover, the variable q^2 representing the invariant mass of the lepton pair, provides a new lever arm to assess systematic uncertainties related to non-factorizable effects.

Finally, let me mention that the QCDF approach can also be applied to 3-body decays $B \rightarrow \pi\pi\pi$ in the kinematic region where each individual dipion mass $m_{ij}^2 = (p_i + p_j)^2$ is sufficiently large, i.e. the three momenta forming a “mercedes-star”-like configuration, see Fig. 1. A systematic theoretical and phenomenological investigation of this idea will be pursued in [24].

3 Flavour Symmetries in QCD

The approximate flavour symmetries (FS) of light quarks in strong interaction dynamics (isospin for u, d quarks, U -spin for d, s quarks, or the full $SU(3)_F$ for u, d, s) have always been a standard tool in understanding hadronic physics. The wealth of experimental data on non-leptonic b - and c -decays nowadays allows to draw conclusions about first-order FS-breaking corrections. In combination with factorization approaches, this can be used to test assumptions about subleading terms in the $1/m_b$ expansion, and in the long run this may also enhance the sensitivity to finding deviations from the SM in these decay modes.

Isospin and $SU(3)_F$ in $B \rightarrow PP$ and $B \rightarrow PV$

The complete set of isospin, U -spin and $SU(3)_F$ relations among the CP asymmetries in B -meson decays to two pseudoscalars (or to one pseudoscalar and one vector meson), together with first-order symmetry-breaking effects, has recently been analyzed in [25] (see also [26, 27]). Comparing with experimental data, the amount of $SU(3)_F$ breaking turns out to be of reasonable size, e.g.

$$\tilde{\Delta} \equiv \frac{\delta_{\text{CP}}[B_d \rightarrow K^+\pi^-] + \delta_{\text{CP}}[B_s \rightarrow K^-\pi^+]}{\delta_{\text{CP}}[B_d \rightarrow K^+\pi^-] - \delta_{\text{CP}}[B_s \rightarrow K^-\pi^+]} = 0.026 \pm 0.106, \quad (7)$$

if the observables are properly normalized, $\delta_{\text{CP}}[i \rightarrow f] = \frac{8\pi m_i^2}{|\vec{p}_{i \rightarrow f}|} \Delta_{\text{CP}}[i \rightarrow f]$. Additional constraints on $SU(3)_F$ breaking can be obtained from certain theory approaches. As an example, the authors of [25] compare two phenomenological approaches based on different treatment of non-factorizable effects. In the so-called “BBNS” approach [8], the combination of amplitude parameters $\alpha_1\alpha_4^c$ determines the numerically dominant source of strong phases and direct CP violation. In the “BPRS” approach [28] the dominant source of non-factorizable effects is expected from charm-penguin contributions. This leads to different correlations between $SU(3)_F$ breaking in BRs and CP asymmetries for $(B_s \rightarrow K^-\pi^+, B_d \rightarrow \pi^-K^+)$ and $(B_d \rightarrow \pi^-\pi^+, B_s \rightarrow K^-K^+)$ which can be confronted with experimental data. Present data are still consistent with both alternatives, but with improved precision in future experiments more decisive conclusions about non-factorizable $SU(3)_F$ -breaking effects in non-leptonic decays will be possible.

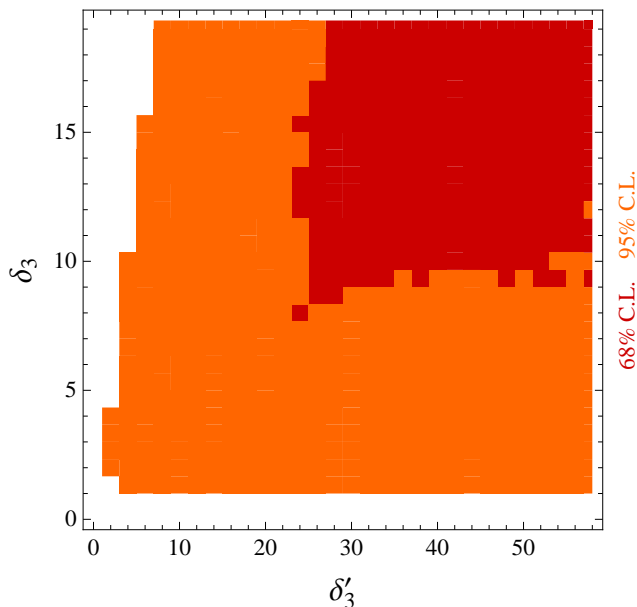


Figure 2: Fit of penguin-enhancement parameters δ_3, δ'_3 to experimental data on non-leptonic charm decays (after CHARM2013). Orange (red) regions indicate the 95% (68%) C.L. regions, see [29] for more details. Fig. taken from [29].

Non-leptonic Charm Decays

Factorization-based approaches to describe exclusive charm decays suffer from the relatively small charm-quark mass, such that non-factorizable corrections are more important than for the corresponding bottom decays. Again, approximate flavour symmetries of QCD turn out to be helpful. The flavour anatomy of D -meson decays into two pseudoscalars has been systematically studied in [29]. Taking into account the complete set of first order $SU(3)_F$ -breaking effects, a consistent fit of the available experimental data could be achieved, with corrections to the flavour-symmetry limit of natural size, $\mathcal{O}(30\%)$. At the time of the analysis, the measured value of the difference between CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays, $\Delta a_{\text{CP}}^{\text{dir}}(K^+K^-, \pi^+\pi^-)$, required a drastic penguin enhancement (in the SM), where also $a_{\text{CP}}^{\text{dir}}(D^0 \rightarrow K_S K_S)$, $a_{\text{CP}}^{\text{dir}}(D_s \rightarrow K_S \pi^+)$, and $a_{\text{CP}}^{\text{dir}}(D_s \rightarrow K^+ \pi^0)$ contribute. After the CHARM2013 conference, the effect is less dramatic, as illustrated in Fig. 2. Alternatively, the $SU(3)_F$ analysis of various CP asymmetries in non-leptonic D -meson decays allows to discriminate between different NP scenarios.

A related $SU(3)_F$ analysis of non-leptonic D -meson decays to two pseudoscalars or one pseudoscalar and one vector meson can be found in [30]. The focus of that work is to derive sum rules among decay amplitudes or decay rates such that $\mathcal{O}(m_s)$

effects drop out. For instance, from the CKM-weighted amplitude relations

$$|(D^0|\pi^+\rho^-)|/\lambda + |(D^0|K^+K^{*-})|/\lambda = |(D^0|K^+\rho^-)|/\lambda^2 + |(D^0|\pi^+K^{*-})| \quad (8)$$

one can infer a prediction for one of the yet unmeasured branching ratios (BRs),

$$\Rightarrow \text{Br}(D^0 \rightarrow \rho^- K^+) \simeq (1.7 \pm 0.4) \cdot 10^{-4}. \quad (9)$$

The formalism can also be combined with the “ $\Delta U = 0$ rule” for large penguins [31] to predict direct CP asymmetries in $D \rightarrow PV$ decays.

Other modes

Recent analyses based on $SU(3)_F$ symmetry have also been performed for particular final state configurations in B -meson decays to three light pseudoscalars [32]; and for 2-body B -meson decays into octet or decuplet baryons [33]. In a systematic study of $SU(3)$ -breaking effects in $B \rightarrow J/\psi P$ decays [34] it has been shown how penguin corrections can be extracted from data, constraining the pollution in the extraction of the CKM angle $\sin 2\beta$ to be very small, $|\Delta S| \leq 0.01$. With more precise data on CP asymmetries and BRs this uncertainty can be further reduced in the future.

4 Hadronic Input Functions

Besides the perturbative computation of short-distance kernels in factorization theorems, and factorization-independent constraints from flavour symmetries, an important role for the quantitative prediction of nonleptonic decays is played by universal hadronic input parameters like decay constants, form factors and light-cone distribution amplitudes (LCDAs). As these contain the information about hadronic binding effects, they have to be determined by nonperturbative methods (i.e. lattice or sum rules) or extracted from experimental data.

Transition Form Factors

If factorization in (1) holds, the form factors for $B \rightarrow M_1$ transitions together with the decay constants f_{M_2} determine the overall magnitude of $B \rightarrow M_1 M_2$ decay amplitudes and BRs at leading order. The sensitivity to this type of hadronic input can be reduced by considering *ratios* with semileptonic decays or among different nonleptonic decays. For theoretical estimates of transition form factors between heavy and light mesons, it has become customary to perform combined fits of light-cone sum-rule results (valid at large energy transfer, see e.g. [35, 36]) and results from QCD simulations on the lattice (valid at low recoil energy, see the discussion in [37] and references given therein) on the basis of the so-called “ z -expansion”.

Light-Cone Distribution Amplitudes for the B -Meson

The B -meson LCDA $\phi_B(\omega)$ determines the size of the spectator interactions in QCDF. The leading term in the short-distance kernels T_i^{II} is proportional to the inverse moment

$$\lambda_B^{-1}(\mu) \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega, \mu), \quad (10)$$

where ω denotes the light-cone projection of the spectator quark's energy. The size of this parameter is crucial for phenomenological analyses. While comparison with data on $B \rightarrow \pi\pi, \pi\rho, \rho\rho$ decays prefers relatively small values around $\lambda_B \sim 200$ MeV within the QCDF/BBNS approach, QCD-sum-rule based estimates typically lead to values in the region $\lambda_B \sim (350 - 500)$ MeV [38]. Recent OPE analyses of the large-scale behaviour of ϕ_B , using the concept of “dual” LCDAs [39] (see also [40]), have shown that – contrary to naive expectation – the parameter λ_B is essentially independent of other HQET parameters like $\bar{\Lambda} = M_B - m_b$ [41]. The most promising approach to independently determine λ_B is to extract its value from experimental data on $B \rightarrow \gamma\ell\nu$ decays, on the basis of QCD factorization theorems and estimates for non-factorizable $1/m_b$ corrections [42, 43]. Presently, using the BaBar bound for that decay rate from 2009, one finds $\lambda_B > 115$ MeV.

Annihilation Parameters in QCDF

Amplitude topologies where the spectator quark in a B -meson annihilates with the b -quark via weak interactions also play a crucial role for the phenomenological analyses of non-leptonic B decays. Notably, these decay topologies cannot be described by heavy-to-light form factors. The $1/m_b$ power corrections induced by annihilation topologies lead to IR-sensitive convolution integrals in QCDF and can thus only be modelled in a crude and ad-hoc manner. In a recent phenomenological analysis of pure annihilation decays of B_d and B_s mesons [44] (see also [45]) the flavour dependence of annihilation parameters in QCDF has been studied. Comparing, on the one hand, $B_d \rightarrow \pi^- K^+$ and $B_s \rightarrow \pi^+ K^-$ decays, one expects similar strong rescattering phases because the final states are related by charge conjugation, which turns out to be in line with experimental observation. On the other hand, comparing $B_s \rightarrow \pi^+ \pi^-$ and $B_d \rightarrow K^+ K^-$ within that approach, sizeable $SU(3)_F$ breaking effects are required.

The size of strong phases in non-leptonic B -meson decays has also been studied within a phenomenological rescattering model [46]. Distinguishing different topological amplitudes (“exchange (E)”, “annihilation (A)”, “penguin-annihilation (PA)”), experimental data reveals a relatively regular pattern, where (E) $\sim (5 - 10)\%$ and (PA) $\sim (15 - 20)\%$ of the largest amplitude from which they can rescatter. This allows one to estimate several BRs for not yet observed B and B_s decays.

5 Summary/Outlook

The dynamics of strong interactions in non-leptonic decays of heavy mesons is extremely complex. While one has to admit that on the theory side a conceptual breakthrough for the systematic calculation of non-factorizable hadronic effects is still lacking, the combination of several theoretical methods in many cases still gives a satisfactory phenomenological picture.

- Short-distance kernels in the QCD factorization approach are now being calculated at NNLO for a variety of decays.
- Systematic studies of $SU(3)_F$ flavour-symmetry breaking effects on the basis of phenomenological data are available.
- The ongoing improvement of the experimental situation leads to better knowledge on hadronic input parameters and more reliable estimates of systematic theoretical uncertainties.

We are thus looking forward to phenomenological updates that combine the state-of-the-art results for radiative corrections, hadronic input parameters, and $SU(3)_F$ -breaking effects.

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